Intermediate Macroeconomics (UN3213)
Recitation 5

Niyuan Huang

March 4, 2025

Table of Contents

Cagan model

- Builds upon QTM by incorporating an interest-elastic demand for money
- Demand for money in the QTM was purely transactive. But in the Cagan model,

  $$M_t^d = L(i_t, Y_t)$$

  which is decreasing in i_t and increasing in Y_t.
  i_t is like the opportunity cost of holding money (interest forgone).
- Recall from the Fisher equation

  $$1 + i_t = (1 + r_t)(1 + \pi_{t+1}^e)$$

  r_t is assumed to be exogenous and constant (not in control of the central bank). The above is approximated as

  $$i_t = r_t + \pi_{t+1}^e$$

Formation of expectations

- **Adaptive expectations:** Expectation is determined by past levels of inflation. Two examples:
  - Depends on only the previous period's inflation

$$\pi_{t+1}^e = \pi_t$$

  - Depends on inflation in all the past periods

$$\pi_{t+1}^e = (1-\beta)(\pi_t + \beta\pi_{t-1} + \beta^2\pi_{t-2} + \dots)$$

  Weights are decreasing and add up to 1.

- **Rational expectations:** People do not commit systematic mistakes:

$$\pi_{t+1}^e = \pi_{t+1}$$

  Actual inflation matches expected inflation when there are no shocks (perfect foresight).

Building blocks of the Cagan model

- Money market equilibrium
$$\frac{M_t}{P_t} = L(i_t, Y_t)$$
- Fisher equation
$$1 + i_t = (1 + r_t)(1 + \pi^e_{t+1})$$
- An assumption about formation of inflation expectations, e.g. whether it is adaptive or rational
- A specification of monetary policy.

Use the above to solve the model, i.e. the equilibrium paths of prices $\{P_t, \pi^e_t, i_t\}_{t=0}^{\infty}$

Guess and verify

- Assume $r_t = r$, $Y_t = Y$, and rational expectations. The equilibrium is described by the following relationships

$$\frac{M_t}{P_t} = Y(1+i_t)^{-\eta}$$
$$1 + i_t = (1+r)(1+\pi_{t+1})$$
$$M_{t+1} = (1+\mu)M_t$$

- **Guess** that M_t/P_t is constant.
- Need to **verify** that the resultant equilibrium satisfies the above three equations.

Guess and verify

- If M_t/P_t is constant P_t must grow at the same rate as M_t. Therefore,

$$1 + \pi_{t+1} \equiv \frac{P_{t+1}}{P_t} = 1 + \mu$$

  or $\pi_{t+1} = \mu$.

- Obtain the path for the nominal interest rate from the Fisher equation.

$$1 + i_t = (1 + r)(1 + \mu)$$

- Obtain the value for M_t/P_t from the money market equilibrium.

$$\frac{M_t}{P_t} = Y[(1 + r)(1 + \mu)]^{-\eta}$$

  which is indeed constant. This verifies our guess is true.
  If you are given the value of M_0, obtain the value for P_0 from the above.

Table of Contents

Fiscal deficit

- The government purchases goods and services (government expenditure), collects taxes (or pays transfers) and pays interest on government debt (debt servicing charges).
- The fiscal deficit is financed by either
  - Printing more money ($M_t - M_{t-1}$)
  - Issuing more debt ($B_t - B_{t-1}$)

  The government's budget constraint is thus

$$\underbrace{P_t G_t - T_t}_{\text{Primary Deficit}} + i_{t-1} B_{t-1} = \underbrace{M_t - M_{t-1}}_{\text{Money Creation}} + \underbrace{B_t - B_{t-1}}_{\text{Debt Issuance}}$$

  All of the terms in the above constraint are in nominal terms.
- The central bank has to be on board for financing through money creation. Debt issuance is in the hands of the Treasury.

Equilibrium

- Divide through by P_t to convert to real terms

$$G_t - \frac{T_t}{P_t} + \frac{i_{t-1}B_{t-1}}{P_t} = \underbrace{\frac{M_t - M_{t-1}}{P_t}}_{\text{Seigniorage}} + \frac{B_t - B_{t-1}}{P_t}$$

- We assume that
  - The fiscal deficit (LHS) stays constant $DEF_t = DEF > 0$, and
  - The deficit is financed exclusively through money creation. The government cannot issue more debt.

$$DEF = \frac{M_t - M_{t-1}}{P_t}$$

- The equilibrium is governed by the money market equilibrium and the Fisher equation. Assume rational expectations.

$$\frac{M_t}{P_t} = L(i_t, y)$$

$$1 + i_t = (1 + r)(1 + \pi_{t+1})$$

Monetary Policy

- Assume money grows at a constant rate μ, i.e.

$$\frac{M_t}{M_{t-1}} = 1 + \mu$$

- We follow the guess-and-verify approach to solve this model
  - Guess $\frac{M_t}{P_t}$ is constant.
  - Therefore the growth rate of P_t (which is inflation π_t) equals μ,
  - From the Fisher equation, $i_t = (1+r)(1+\mu) - 1$ and is constant.
  - From the money market equilibrum, $\frac{M_t}{P_t}$ is constant which verifies our guess.

- Using this solution, we solve for the level of the fiscal deficit.

$$\begin{aligned} DEF &= \frac{M_t}{P_t} - \frac{M_{t-1}}{P_t} \\ &= \frac{M_t}{P_t} - \frac{1}{1+\pi_t}\frac{M_{t-1}}{P_{t-1}} \\ &= L\left((1+r)(1+\mu) - 1, y\right) \cdot \left[1 - \frac{1}{1+\mu}\right] \end{aligned}$$

Inflation tax and the Laffer curve

- For the fiscal deficit to be exactly funded by money growth:

$$DEF = L\left((1+r)(1+\mu)-1, y\right) \cdot \frac{\mu}{1+\mu}$$

- $\mu/(1+\mu)$ acts like an inflation tax rate, which is increasing in μ
- The tax base $L((1+r)(1+\mu)-1, y)$ is decreasing in μ
- The net tax revenue (as a function of μ) can be inverted U-shaped

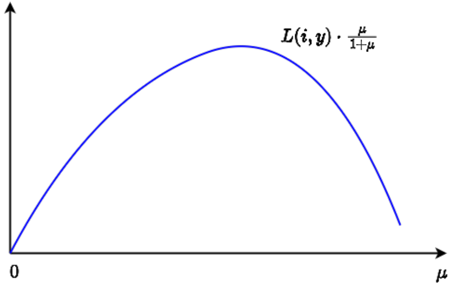
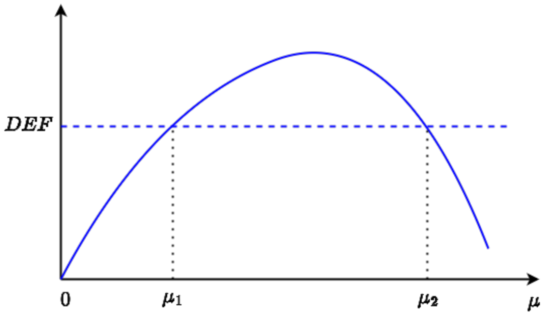


Figure: Laffer curve

Solving for the money growth rate

- To solve for the exact money growth rate to finance the deficit, solve for μ from the below

$$DEF = L\left((1+r)(1+\mu)-1, y\right)\cdot\frac{\mu}{1+\mu}$$



- This results in two solutions μ_1, μ_2. Inflation is higher at mu_2 (recall $\pi_t = \mu$). We assume that the government can implement $\mu = \mu_1$, i.e. we only focus on the upward-sloping part of the Laffer curve.

Size of deficit

- Higher deficit $\rightarrow$ Higher money growth rate needed to finance it

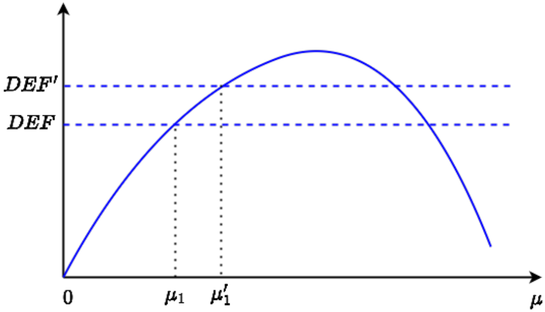


Table of Contents

A numerical example

- Suppose the money demand function is

$$L(i_t, Y) = \gamma \left(\frac{1 + i_t}{i_t} \right) Y$$

where $\gamma = 0.5$, $r = 0.05$,

- The fiscal deficit is a third of GDP, i.e. $DEF = \frac{1}{3} Y$
- At equilibrium (where the deficit is completely financed by money growth)

$$\begin{aligned} DEF &= \frac{\mu}{1+\mu} \cdot \gamma \left(\frac{1 + i_t}{i_t} \right) Y \\ &= \frac{\mu}{1+\mu} \cdot \gamma \frac{(1+r)(1+\mu)}{(1+r)(1+\mu) - 1} \cdot Y \end{aligned}$$

It can be verified that the solution to the above is

$$\mu = \frac{r}{1+r} \cdot \frac{DEF/(\gamma Y)}{1 - DEF/(\gamma Y)} = \frac{0.05}{1+0.05} \cdot \frac{1/3 \cdot 1/2}{1 - 1/3 \cdot 1/2} = 0.0952$$

Inflation is 9.52% and nominal interest rate is 14.7% (from the Fisher eqn).

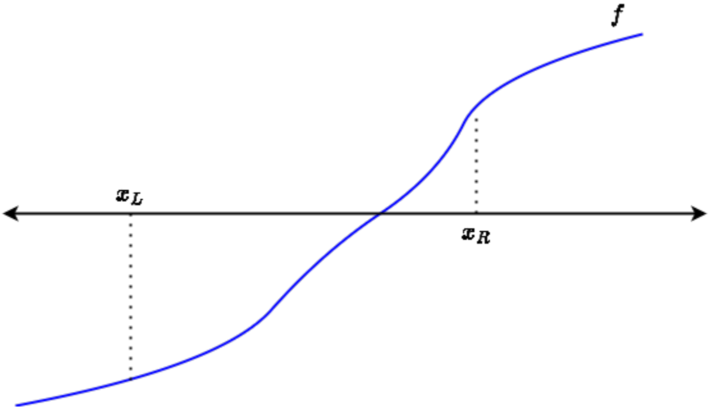
Solution algorithm

We want to solve the equation $f(x) = 0$ where f is some continuous function. Suppose there are two values $x_L < x_R$ such that $f(x_L) < 0$ and $f(x_R) > 0$.

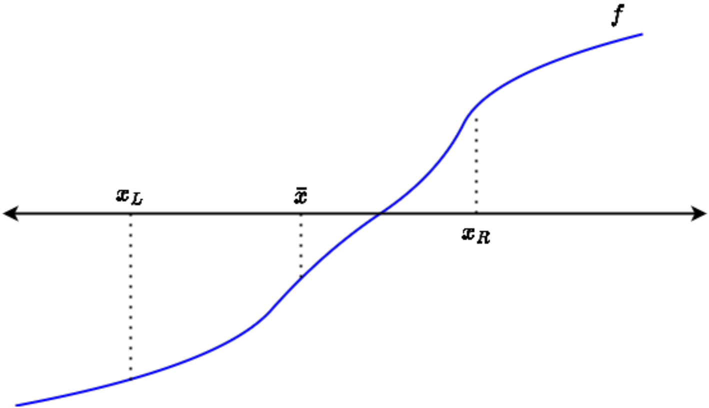
(a) Calculate $\bar{x} = \frac{x_L + x_R}{2}$
(b) Calculate $f(\bar{x})$.
    - If $f(\bar{x}) < 0$, redefine $x_L = \bar{x}$ but keep x_R unchanged,
    - If $f(\bar{x}) > 0$, redefine $x_R = \bar{x}$ but keep x_L unchanged.
(c) Go to step (a).

Repeat the above until we are sufficiently close to the solution (i.e. x_L and x_R are close together).

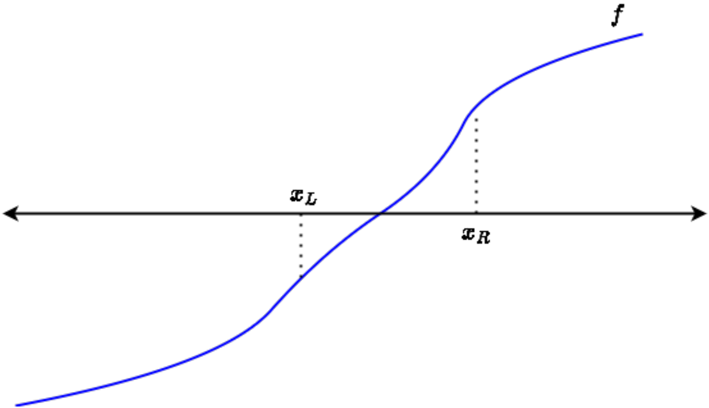
Solution algorithm (visualized)



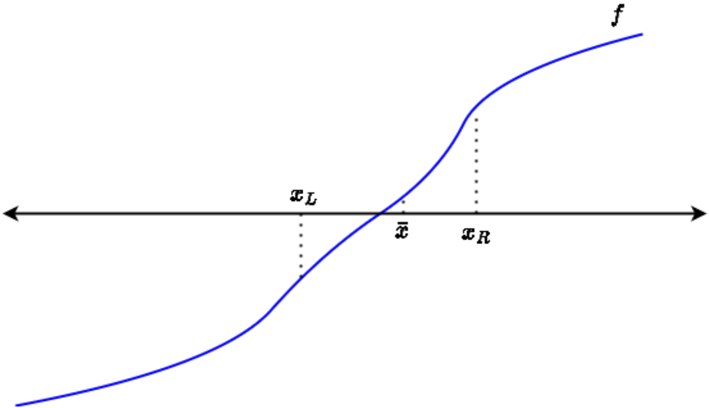
Solution algorithm (visualized)



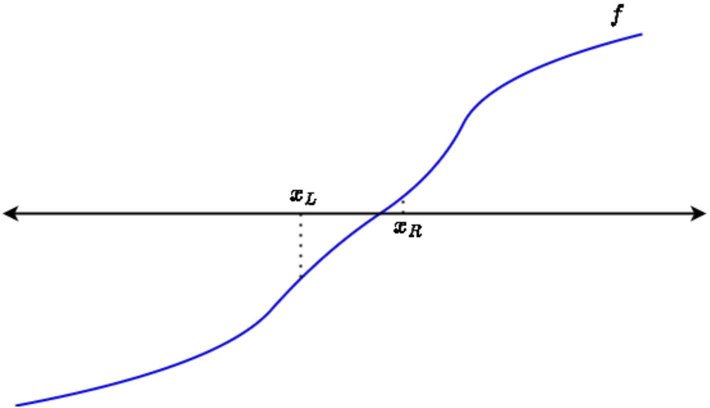
Solution algorithm (visualized)



Solution algorithm (visualized)



Solution algorithm (visualized)



Implementation

Back to the deficit financing problem:

- Define the function $g(\mu)$.

$$g(\mu) = \frac{\mu}{1+\mu} \cdot \gamma \frac{(1+r)(1+\mu)}{(1+r)(1+\mu)-1} - \frac{DEF}{Y}$$

  where $\frac{DEF}{Y} = \frac{1}{3}$.
  Need to solve $g(\mu) = 0$.

- Take $\mu_L = 0$ and $\mu_R = 1$. Clearly $g(\mu_L) < 0$, and $g(\mu_H) > 0$, given the values of the other parameters in the problem.

Follow the rest of the implementation in `deficit_computations.xlsx`