Intermediate Macroeconomics (UN3213)
Recitation 7

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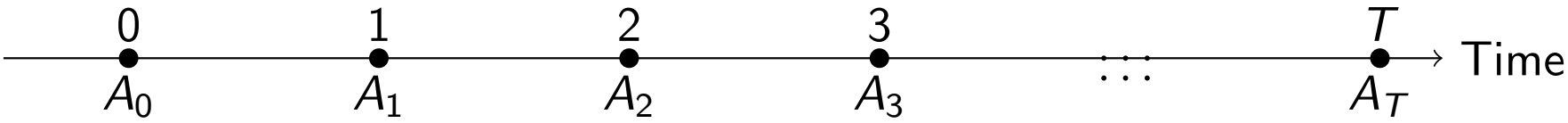
Present and future values

- Consider an asset that yields cashflows $A_0, A_1, A_2, \ldots A_T$ over $T+1$ time periods

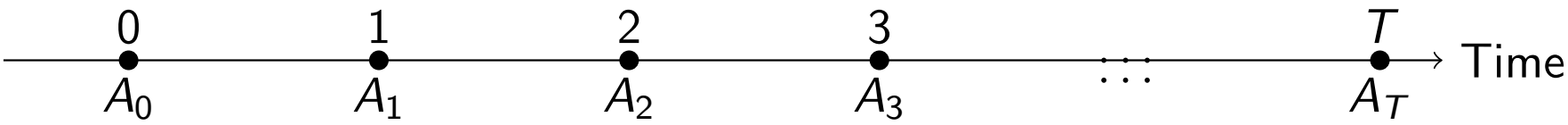


- Investors can alternatively put their savings in a bank which pays a risk-free rate r.
- **Future value** is the value of the stream of cashflows evaluated at future date $t = T$

$$\begin{aligned} FV_T &= A_0(1+r)^T + A_1(1+r)^{T-1} + \cdots + A_{T-1}(1+r)^1 + A_T(1+r)^0 \\ &= \sum_{t=0}^{T} A_t(1+r)^{T-t}. \end{aligned}$$

Present and future values

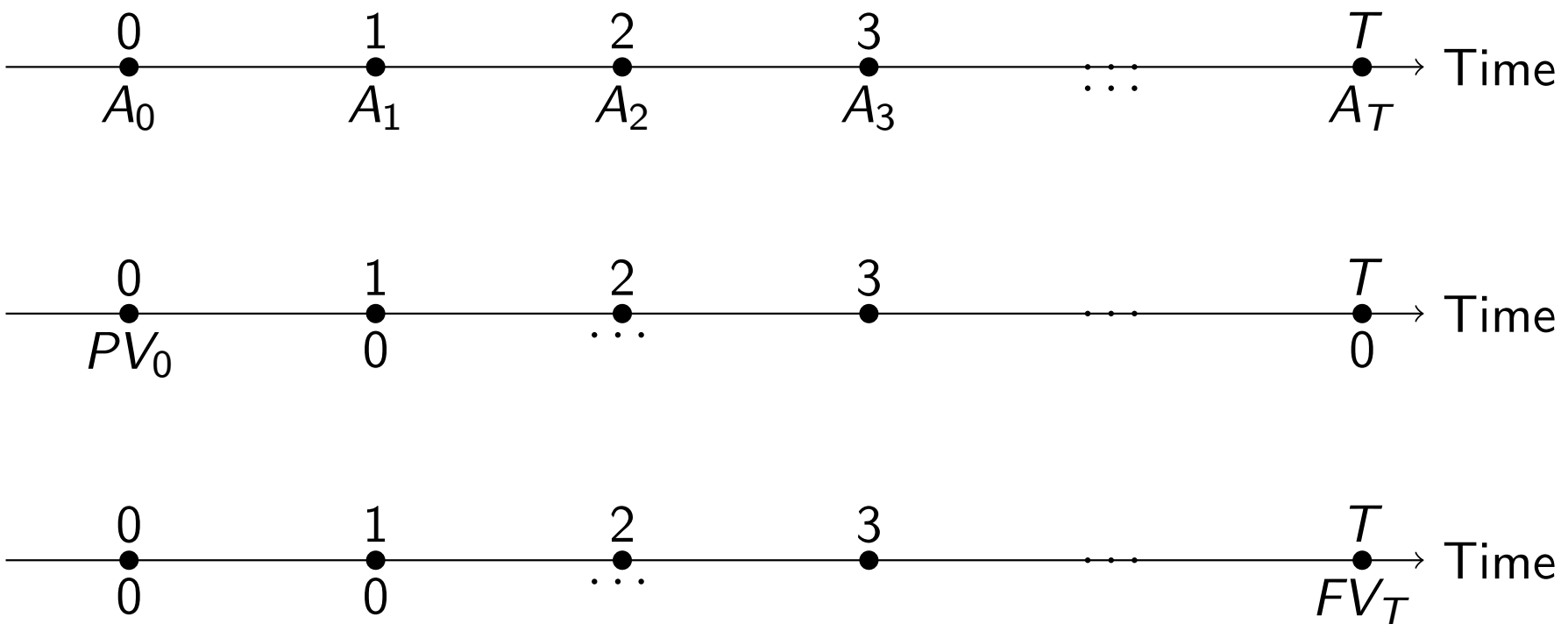
- Consider an asset that yields cashflows $A_0, A_1, A_2, \ldots A_T$ over $T+1$ time periods



- Investors can alternatively put their savings in a bank which pays a risk-free rate r.
- **Present value** is the value of the stream of cashflows evaluated at current date $t = 0$

$$PV_0 = A_0 \frac{1}{(1+r)^0} + A_1 \frac{1}{(1+r)^1} + \cdots + A_{T-1} \frac{1}{(1+r)^{T-1}} + A_T \frac{1}{(1+r)^T}$$
$$= \sum_{t=0}^{T} A_t \frac{1}{(1+r)^t}.$$

Equivalent cashflows



The above three streams of cashflows are all equivalent. Also note that

$$FV_T = (1 + r)^T PV_0$$

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Two-period model

- The government's budget constraint equates PV of government expenditure stream to the PV of tax revenue collected.

$$G_0 + \frac{G_1}{1+r} = T_0 + \frac{T_1}{1+r}$$

  This is a *consolidated (intertemporal) budget constraint*.

- Where is this coming from? $\rightarrow$ Aggregattion of two *per-period budget constraints*.

- Budget constraints for each period:

$$\begin{aligned} t = 0: &\quad G_0 + (1+r)B_0 - T_0 = B_1 \\ t = 1: &\quad G_1 + (1+r)B_1 - T_1 = 0 \end{aligned}$$

  We are imposing that government has to pay back all debt at $t = 1$ (cannot run a Ponzi scheme)

- Eliminate B_1 from the two equations and assume $B_0 = 0$ to derive the intertemporal government budget constraint

Ricardian equivalence

- Household maximizes lifetime discounted utility

$$\max_{\{c_0, c_1, s\}} u(c_0) + \beta u(c_1)$$

subject to the budget constraints for each period

$$\begin{aligned} t = 0: &\quad y_0 - c_0 - T_0 = s \\ t = 1: &\quad (1+r)s + y_1 - T_1 = c_1 \end{aligned}$$

y_0 and y_1 are incomes in each period (exogenous and constant).

- Eliminating s derive the household's intertemporal budget constraint

$$c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r} - \left(T_0 + \frac{T_1}{1+r} \right)$$

Ricardian equivalence

- The government's intertemporal budget constraint has to hold

$$G_0 + \frac{G_1}{1+r} = T_0 + \frac{T_1}{1+r}$$

- Therefore the household's intertemporal budget constraint can be re-written as

$$c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r} - \left(G_0 + \frac{G_1}{1+r} \right)$$

  Note that G_0, G_1 are exogenous and constant. The right hand side does not depend on T_0, T_1. Therefore a tax cut at $t = 0$ do not affect the household's budget constraint or their consumption choice (demand).

- Since government expenditure G_0, G_1 are fixed, from the government budget constraint

$$\Delta T_0 = -\frac{\Delta T_1}{1+r}$$

  Any tax cut at $t = 0$ has to be accompanied by a tax hike at $t = 1$.