Intermediate Macroeconomics (UN3213)
Recitation 8

Niyuan Huang

April 1, 2025

Table of Contents

Fiscal policy

- Do tax cuts stimulate the economy? What is its effect on real economic variables (c, I, r)?
- We want to write down a simple two-period model to answer this question, which consists of the following agents:
  - Government
  - Firms
  - Households

Table of Contents

Some definitions

Let G_t denote government expenditure in period t, T_t is the tax revenue raised, and B_t real government debt issued.

- Difference between government expenditure and tax revenue

$$\textbf{Primary fiscal deficit} = G_t - T_t$$

- Difference between total expenses (government expenditure and interest payments) and tax revenue

$$\textbf{Secondary fiscal deficit} = G_t - T_t + r_{t-1}B_{t-1}$$

- Negative of the secondary fiscal deficit

$$\textbf{Government saving} = T_t - G_t - r_{t-1}B_{t-1}$$

Note that $B_t > 0$ denotes borrowing, and $B_t < 0$ denotes lending.

A two-period example

- Two periods $t = 1, 2$. the government starts period 1 with B_0 outstanding debt
- Period 1 budget constraint:

$$G_1 + (1 + r_0)B_0 - T_1 = B_1$$

Rewriting this, the secondary fiscal deficit has to be financed by issuing new debt, i.e.

$$\underbrace{G_1 + r_0 B_0 - T_1}_{\text{secondary fiscal deficit}} = \underbrace{B_1 - B_0}_{\text{new debt}}$$

- Period 2 budget constraint:

$$G_2 + (1 + r_1)B_1 - T_2 = 0$$

Rewriting,

$$G_2 + r_1 B_1 - T_2 = -B_1$$

We impose that the government has to pay back all debt in full at the end of period 2.

Intertemporal budget constraint

- Assume that $B_0 = 0$ (no initial outstanding debt). The budget constraints for each period are

$$G_1 - T_1 = B_1$$
$$G_2 + (1 + r_1)B_1 - T_2 = 0$$

- Eliminate B_1 from the above two equations to derive the intertemporal (consolidated) budget constraint.

$$G_2 + (1 + r_1)(G_1 - T_1) - T_2 = 0$$
$$\Longrightarrow G_1 + \frac{G_2}{1 + r_1} = T_1 + \frac{T_2}{1 + r_1}$$

Present value of government expenditure equals present value of tax revenue.

HW5: Q3 (1)

Consider a 2-period economy. The initial public debt is zero ($B_0 = 0$). The primary deficit is zero in both periods. This government satisfies its intertemporal budget constraint.

HW5: Q3 (1)

Consider a 2-period economy. The initial public debt is zero ($B_0 = 0$). The primary deficit is zero in both periods. This government satisfies its intertemporal budget constraint. Primary deficit is zero in both periods, i.e.

$$G_1 - T_1 = 0$$
$$G_2 - T_2 = 0$$

The intertemporal budget constraint can be rewritten as follows

$$G_1 + \frac{G_2}{1+r_1} = T_1 + \frac{T_2}{1+r_1}$$
$$\Longleftrightarrow (G_1 - T_1) + \frac{1}{1+r_1}(G_2 - T_2) = 0$$

The LHS is zero if primary deficit is zero in both periods. The intertemporal budget constraint is satisfied. **TRUE**

HW5: Q3 (2)

Consider a 2-period economy. The initial public debt is zero ($B_0 = 0$). The government runs primary surpluses in both periods. This government does not satisfy its intertemporal budget constraint.

HW5: Q3 (2)

Consider a 2-period economy. The initial public debt is zero ($B_0 = 0$). The government runs primary surpluses in both periods. This government does not satisfy its intertemporal budget constraint. Use the same form of the

intertemporal budget constraint as before:

$$(G_1 - T_1) + \frac{1}{1 + r_1}(G_2 - T_2) = 0$$

If the government runs a primary fiscal surplus in both periods,

$$G_1 - T_1 < 0$$
$$G_2 - T_2 < 0$$

Clearly, the intertemporal budget constraint does not hold because the LHS < 0.
TRUE

Ricardian equivalence

- The government's intertemporal budget has to hold

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

- G_0 and G_1 are exogenous and constant. Therefore the LHS is constant.
- Any change in taxes in one period has to be offset by an opposite change in taxes in the other period.

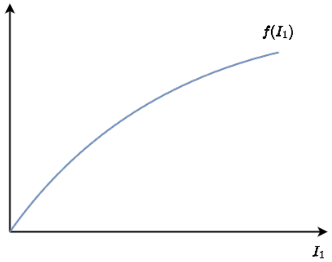
$$\Delta T_1 = -\frac{1}{1+r}\Delta T_2$$

A cut in taxes does not affect the consumption-saving decisions of households since they anticipate a rise in taxes in the future.

Table of Contents

Production

- In period 1, firms undertake investment (I_1). They borrow money to build capital.
- In period 2, the firm
  - Produces output, $y = f(I_1)$,
  - Pays back debt from period 1,
  - Distributes profits to households (e.g. dividends)
- Production function is strictly increasing and concave (exhibits diminishing returns to capital)



Profit maximization

- The objective of the firms is to maximize profit (revenue minus cost of capital borrowed)

$$\pi_2 = f(I_1) - (1 + r_1)I_1$$

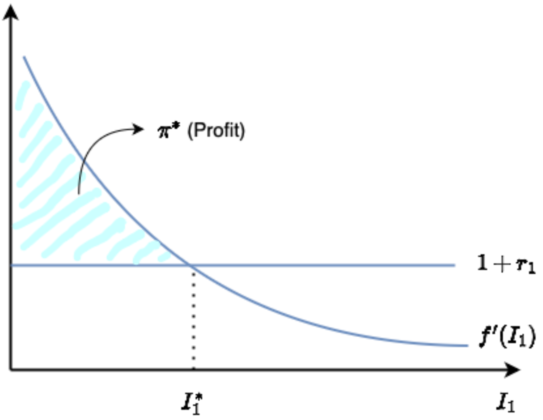
  Note that price of output is normalized to 1.

- First order condition:

$$\frac{\partial \pi_2}{\partial I_1} = 0 \implies f'(I_1) = (1 + r_1)$$

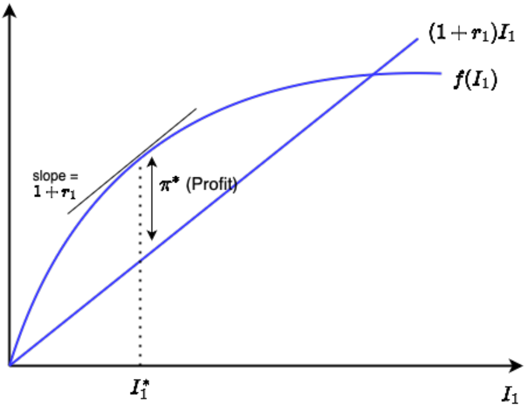
- Second order condition is satisfied due to concavity of production function, $f''(I_1) < 0$

Profit maximization visualized



- Total revenue is $f(I_1^*) = \int_0^{I_1^*} f'(i)di$
- Total cost of capital is $(1 + r_1)I_1^*$

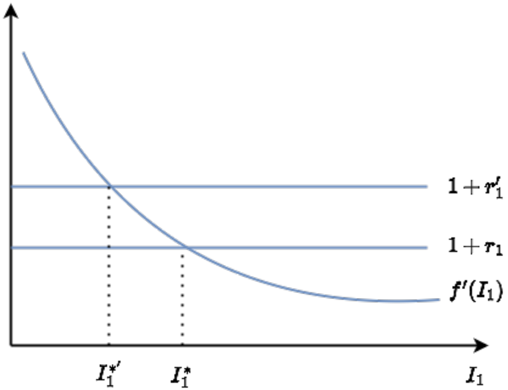
Profit maximization: Alternative interpretation



- Maximized profit is the gap between the production curve and the cost of capital at the point where their slopes are equal (i.e. $f'(I_1^*) = 1 + r_1$)

Cost of capital

- Higher cost of borrowing $\implies$ Less capital accumulation

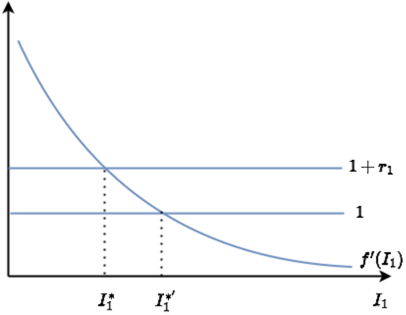


HW5 Q3 (3)

If the real interest rate in period 1 is zero ($r_1 = 0$), then the firm's optimal level of investment in period 1 is infinite ($I_1 \to \infty$).

HW5 Q3 (3)

If the real interest rate in period 1 is zero ($r_1 = 0$), then the firm's optimal level of investment in period 1 is infinite ($I_1 \to \infty$). The cost per unit of capital is the gross interest rate $1 + r_1$ which is always positive.



If $r_1 = 0$, first order condition is $f'(I_1^{*'}) = 1$, i.e. $I_1^{*'} = f'^{-1}(1)$ which is still finite.
FALSE

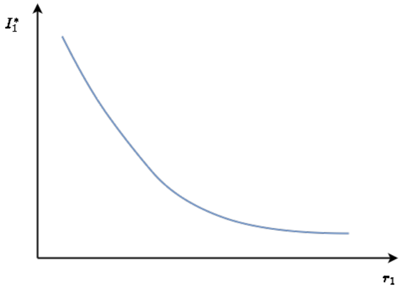
Investment schedule

- Profit-maximizing behavior of firms gives rise to a relation between optimal investment and interest rates. From the first order condition:

$$f'(I_1^*) = 1 + r_1$$
$$\implies I_1^* = f'^{-1}(1 + r_1) \equiv I(r_1)$$

  $I(r_1)$ is called the investment schedule.

- It is a downward-sloping relation



Profits and interest rate

- What happens to profit as the interest rate rises?
- Write down the profit function in terms of investment and differentiate.

  $$\pi(r_1) \equiv f(I(r_1)) - (1+r_1)I(r_1)$$

  Differentiating,

  $$\begin{aligned} \pi'(r_1) &= f'(I(r_1))I'(r_1) - I(r_1) - (1+r_1)I'(r_1) \\ &= [f'(I(r_1) - (1+r_1)] \cdot I'(r_1) - I(r_1) \\ &= -I(r_1) < 0 \end{aligned}$$

  where the second inequality follows because of the first order condition $f'(I(r_1) - (1+r_1)$.
- Profit is decreasing in interest rate.

Table of Contents

Utility maximization

- Household maximizes lifetime discounted utility

$$\max_{\{C_1, C_2, S_1^P\}} \ln(C_1) + \ln(C_2)$$

subject to the budget constraints for each period

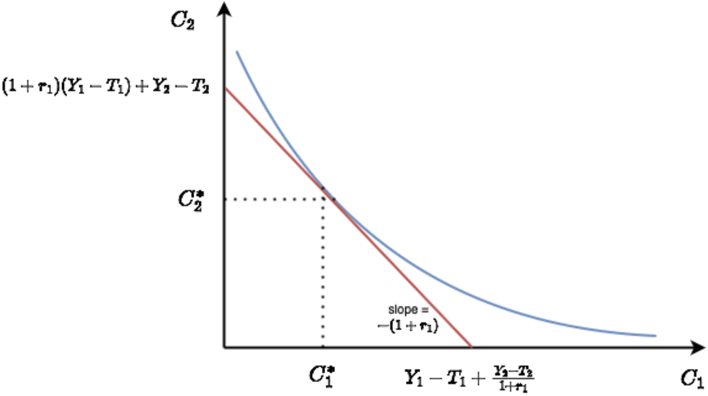
$$t = 1: \quad Y_1 - C_1 - T_1 = S_1^P$$
$$t = 2: \quad (1 + r_1)S_1^p + Y_2 - T_2 = C_2$$

Y_1 and Y_2 are incomes in each period (exogenous and constant).

- Eliminating s derive the household's intertemporal budget constraint

$$C_1 + \frac{C_2}{1 + r_1} = Y_1 - T_1 + \frac{Y_2 - T_2}{1 + r_1}$$

Utility maximization visualized



- Indifference curves are
  - Strictly decreasing
  - Convex
  - Non-intersecting

Optimal consumption choice

- Rewriting the household's problem as

$$\max_{\{C_1, C_2\}} \ln(C_1) + \ln(C_2)$$

subject to the intertemporal budget constraint

$$C_1 + \frac{C_2}{1 + r_1} = Y_1 - T_1 + \frac{Y_2 - T_2}{1 + r_1}$$

- Use the budget constraint to substitute out one of the variables.

$$\max_{C_1} \quad \ln(C_1) + \ln\left((1 + r_1)(Y_1 - T_1 - C_1) + (Y_2 - T_2)\right)$$

- First order condition

$$\frac{1}{C_1} - \frac{(1 + r_1)}{(1 + r_1)(Y_1 - T_1 - C_1) + (Y_2 - T_2)} = 0$$

Optimal consumption choice

- Optimal consumption in period 1:

$$C_1^* = \frac{1}{2}\left[Y_1 - T_1 + \frac{Y_2 - T_2}{1 + r_1}\right]$$

- Optimal consumption in period 2:

$$C_1^* = \frac{1}{2}(1 + r_1)\left[Y_1 - T_1 + \frac{Y_2 - T_2}{1 + r_1}\right]$$

- Optimal private saving:

$$\begin{aligned} S_1^{p*} &= Y_1 - T_1 - C_1^* \\ &= \frac{1}{2}\left[Y_1 - T_1 - \frac{Y_2 - T_2}{1 + r_1}\right] \end{aligned}$$