Intermediate Macroeconomics (UN3213)
Recitation 9

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(1.1) We have $G_1 = 3$, $G_2 = 1$, $T_2 = 2.1$, $r_1 = 0.1$

Usual assumptions: $B_0 = 0$, $B_2 = 0$

IGBC is $$G_1 + \frac{G_2}{1+r_1} = T_1 + \frac{T_2}{1+r_1}$$

$$\Rightarrow 3 + \frac{1}{1.1} = T_1 + \frac{2.1}{1.1}$$

$$\Rightarrow T_1 = 3 + \cancel{\frac{1-2.1}{1.1}}^{\,-1} = 2$$

$$\Rightarrow \boxed{PDEF_1 = SDEF_1 = G_1 - T_1 = 1}$$

(1.2) Since $B_0 = 0$, we have $B_1 = G_1 - T_1 = 1$

This means the govt borrow 1 unit in $t=1$. It will have to repay 1.1 units in $t=2$. This is exactly the amount of money it plans to collect in excess of spending in $t=2$.

1.3, 1.4

(1.3) $SDEF_2 = G_2 - T_2 + r_1 B_1$

$= 1 - 2.1 + 0.1(1)$

$= -1$

Note that $SDEF_1 + SDEF_2 = 0$!

All fiscal deficit across time must sum to 0

(1.4) If $B_1 = 0$ (balance deficit in $t=1$) $\Rightarrow$ $T_1 = G_1 = 3$

In $t=2$, it follows $(1+r_1)B_1 = 0 = T_2 - G_2 \Rightarrow T_2 = G_2 = 1$

$\Rightarrow SDEF_2 = 0$

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Utility maximization

- Household maximizes lifetime discounted utility

$$\max_{\{C_1, C_2, S_1^P\}} \ln(C_1) + \ln(C_2)$$

subject to the budget constraints for each period

$$\begin{aligned} t = 1: &\quad Y_1 - C_1 - T_1 = S_1^P \\ t = 2: &\quad (1 + r_1)S_1^p + Y_2 - T_2 = C_2 \end{aligned}$$

Y_1 and Y_2 are household incomes in each period.
$S_1^P > 0$ denotes saving, $S_1^P < 0$ denotes borrowing.

- Eliminate S_1^P and derive the household's intertemporal budget constraint

$$C_1 + \frac{C_2}{1 + r_1} = Y_1 - T_1 + \frac{Y_2 - T_2}{1 + r_1}$$

Present value of lifetime consumption equals present value of lifetime disposable income.

Utility maximization

- Rewriting the households' problem

$$\max_{\{C_1, C_2\}} \ln(C_1) + \ln(C_2)$$

subject to the intertemporal budget constraint

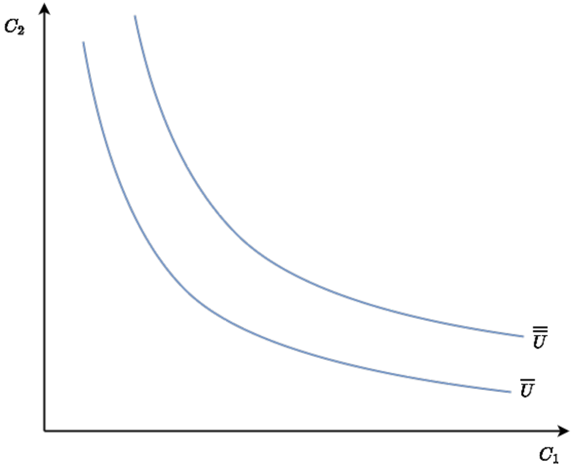
$$C_1 + \frac{C_2}{1+r_1} = Y_1 - T_1 + \frac{Y_2 - T_2}{1+r_1}$$

- The two goods here are consumption in period 1 and consumption in period 2.
- Rewriting the intertemporal budget constraint in future value terms (multiplying both sides by $1 + r_1$):

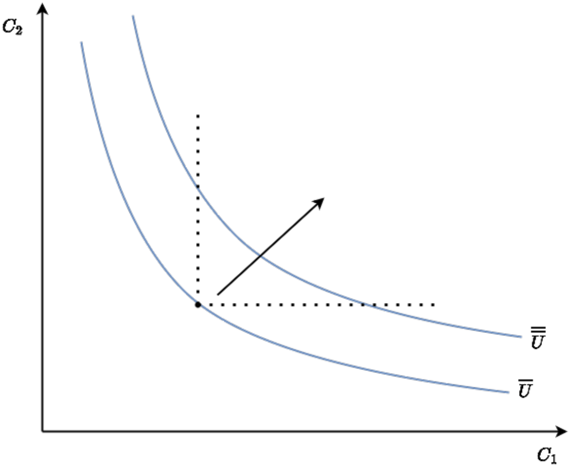
$$(1+r_1)C_1 + C_2 = (1+r_1)(Y_1 - T_1) + (Y_2 - T_2)$$

Price of C_1 relative to C_2 is $1 + r_1$. This is the opportunity cost of consuming one more unit in period 1, in terms of consumption forgone in period 2.

Indifference curves

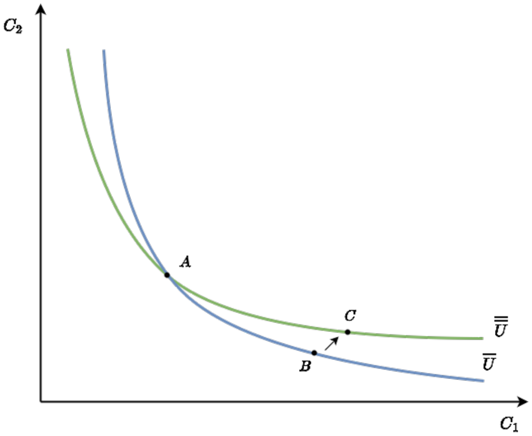


C_1, C_2 are 'goods'



Utility is strictly higher in the north-east quadrant, $\overline{\overline{U}} > \overline{U}$

Indifference curves cannot intersect



Consumer must be indifferent between A and B, and between A and C. This implies they must be indifferent between B and C. But since C offers more of each good compared to B, they must be happier consuming C than B (contradiction!).

Indifference curves are convex

- Consider some utility function $U(C_1, C_2)$. An indifference curve is represented as

$$U(C_1, C_2) = \overline{U}$$

Since the level of utility is constant, total derivative of U should be zero.

$$\frac{\partial U(C_1, C_2)}{\partial C_1} \cdot dC_1 + \frac{\partial U(C_1, C_2)}{\partial C_2} \cdot dC_2 = 0$$

$$\iff \left. \frac{dC_2}{dC_1} \right|_{U=\overline{U}} = -\frac{\partial U(C_1, C_2)/\partial C_1}{\partial U(C_1, C_2)/\partial C_2} \equiv -\frac{MU_{C_1}}{MU_{C_2}}$$

Slope of the indifference curve is the ratio of the marginal utilities at that point.

- Applying to our example, $U(C_1, C_2) = \ln(C_1) + \ln(C_2)$,

$$\left. \frac{dC_2}{dC_1} \right|_{U=\overline{U}} = -\frac{1/C_1}{1/C_2} = -\frac{C_2}{C_1}$$

ICs are convex because of **diminishing marginal utility**.

Solution method

- Recall the household's optimization problem:

$$\max_{\{C_1, C_2\}} \ln(C_1) + \ln(C_2)$$
$$\text{s.t.} \quad (1 + r_1)C_1 + C_2 = \overline{Y}$$

where $\overline{Y} = (1 + r_1)(Y_1 - T_1) + (Y_2 - T_2)$

- Use the budget constraint to eliminate one of the two variables:

$$\max_{C_1} \ln(C_1) + \ln\left(\overline{Y} - (1 + r_1)C_1\right)$$

First-order condition:

$$\frac{1}{C_1^*} - \frac{1 + r_1}{\overline{Y} - (1 + r_1)C_1^*} = 0$$
$$\iff C_1^* = \frac{1}{2} \cdot \frac{\overline{Y}}{1 + r_1}$$

Optimal consumption bundle

- Optimal consumption in period 1:

$$C_1^* = \frac{1}{2} \cdot \frac{\overline{Y}}{1+r_1} = \frac{1}{2}\left[Y_1 - T_1 + \frac{Y_2 - T_2}{1+r_1}\right]$$

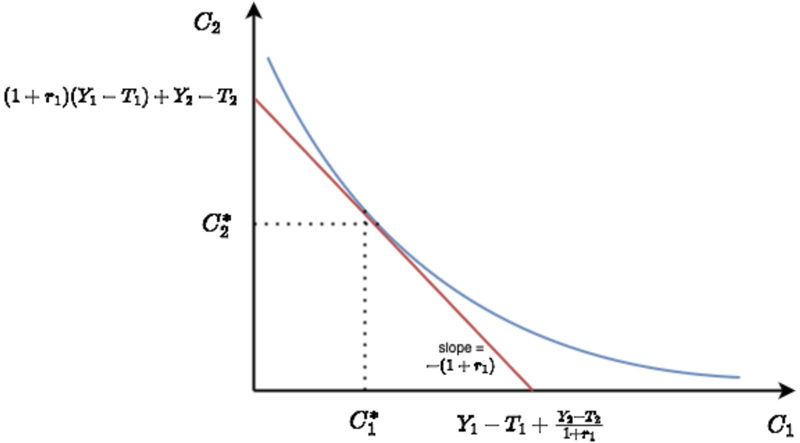
- Optimal consumption in period 2:

$$C_2^* = \frac{1}{2} \cdot \overline{Y} = \frac{1}{2}(1+r_1)\left[Y_1 - T_1 + \frac{Y_2 - T_2}{1+r_1}\right]$$

- Optimal private saving:

$$\begin{aligned} S_1^{p*} &= Y_1 - T_1 - C_1^* \\ &= \frac{1}{2}\left[Y_1 - T_1 - \frac{Y_2 - T_2}{1+r_1}\right] \end{aligned}$$

Utility maximization visualized



Alternative method

- At optimal consumption, slope of the indifference curve is equal to slope of budget line.

$$-\frac{C_2}{C_1} = -(1 + r_1)$$

- The budget constraint has to hold.

$$(1 + r_1)C_1 + C_2 = \overline{Y}$$

- Solve for C_1^*, C_2^* from these two equations.

$$C_1^* = \frac{1}{2} \cdot \frac{\overline{Y}}{1 + r_1}, \quad C_2^* = \frac{1}{2} \cdot \overline{Y}$$

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National saving

- Private saving arises from households' intertemporal consumption choice:

$$S_1^p = Y_1 - T_1 - C_1$$

- Government saving is given by the fiscal surplus in period 1:

$$\begin{aligned} S_1^g &= T_1 - (1 + r_0)B_0 - G_1 \\ &= T_1 - G_1 \end{aligned}$$

We assume $B_0 = 0$.

- National saving is the sum of the two:

$$\begin{aligned} S_1 &= S_1^p + S_1^g \\ &= (Y_1 - T_1 - C_1) + (T_1 - G_1) \\ &= Y_1 - G_1 - C_1 \\ &= Y_1 - G_1 - \frac{1}{2} \cdot \left[Y_1 - T_1 + \frac{Y_2 - T_2}{1 + r_1} \right] \end{aligned}$$

Saving schedule

- Recall that the government has to maintain its intertemporal budget constraint over the two periods:

$$T_1 + \frac{T_2}{1+r_1} = G_1 + \frac{G_2}{1+r_1}$$

- Plugging this into the expression for national saving:

$$\begin{aligned} S_1 &= Y_1 - G_1 - \frac{1}{2} \cdot \left[Y_1 - G_1 + \frac{Y_2 - G_2}{1+r_1} \right] \\ &= \frac{1}{2} \cdot \left[Y_1 - G_1 - \frac{Y_2 - G_2}{1+r_1} \right] \end{aligned}$$

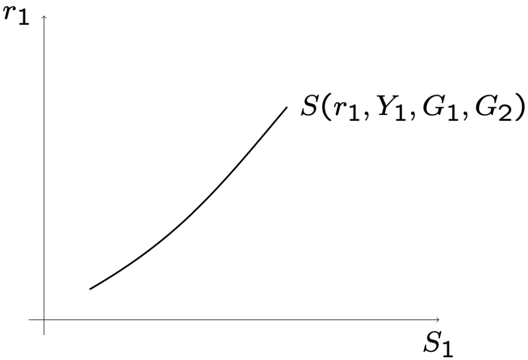
- Y_1 is constant and exogenous, but recall that the firms' profit in period 2 is disbursed to households. Therefore, $Y_2 = \Pi_2(r_1)$. Therefore,

$$S_1 = \frac{1}{2} \cdot \left[Y_1 - G_1 - \frac{\Pi_2(r_1) - G_2}{1+r_1} \right] \equiv S_1(r_1; Y_1, G_1, G_2)$$

Saving schedule

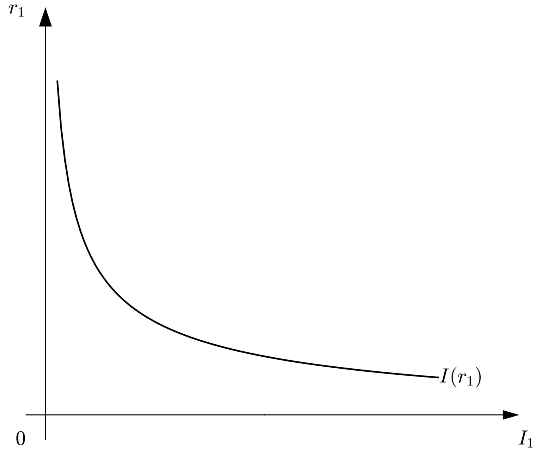
- Recall that the profit function $\Pi()$ is decreasing in r_1. Therefore the term $\frac{\Pi_2(r_1)-G_2}{1+r_1}$ is decreasing in r_1. Therefore,

$$S_1 = S\left(\underset{+}{r_1}; \underset{+}{Y_1}, \underset{-}{G_1}, \underset{+}{G_2}\right)$$



Investment schedule

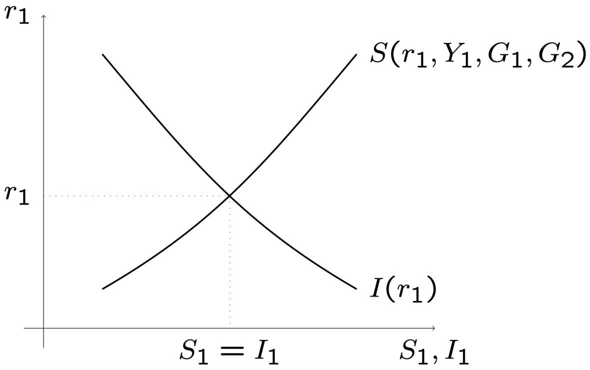
- Recall from the firms' optimization that optimal investment was decreasing in the interest rate, $I'(r_1) < 0$.



Equilibrium

- Solve for the equilibrium interest rate r_1^* from the saving and investment schedules.

$$S(r_1^*; Y_1, G_1, G_2) = I(r_1^*)$$



- **Ricardian equivalence:** Change in taxes has no effect on equilibrium.

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Unrealistic assumptions

The Ricardian equivalence result breaks down when the following assumptions are relaxed:

(a) No borrowing constraints

(b) Infinite lives

(c) Lump-sum taxes

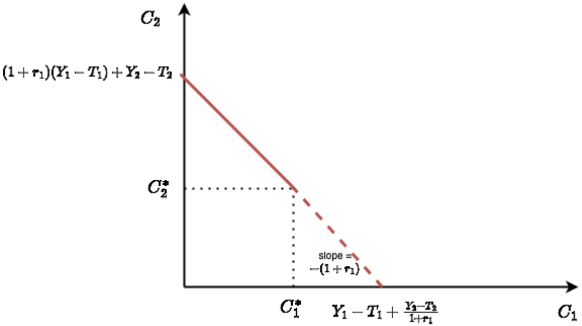
Borrowing constraints

- Recall the period 1 budget constraint for the household:

$$C_1 + S_1^P = Y_1 - T_1$$

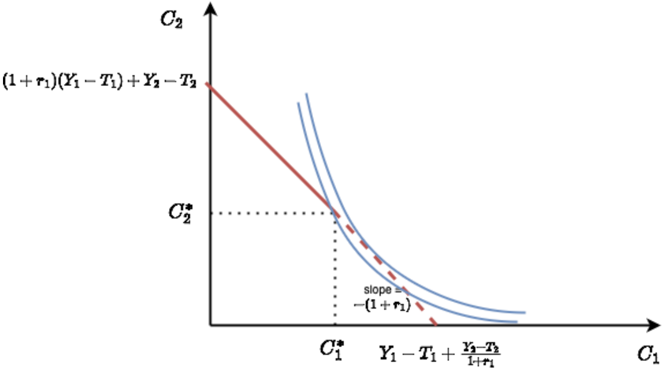
  The borrowing constraint is $S_1^P \geq 0$

- The intertemporal budget constraint is truncated:



Optimal choice under constraint

- Because of the constraint, the household cannot borrow even if it wants to. They are forced to consume their endowment: $C_1^* = Y_1 - T_1$, $C_2^* = Y_2 - T_2$.



Effect of tax cut

- When households are borrowing-constrained, they consume their endowment, and do not save, i.e. $S_1^p = Y_1 - T_1 - C_1 = 0$.
- All savings come from the government, $S_1 = S_1^g = T_1 - G_1$. Saving is interest-inelastic in this case.
- When there is a tax cut, saving contracts: $\Delta S_1 = \Delta T_1$