Intermediate Macroeconomics (UN3213)
Recitation 10

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Q1

- Firms maximize profit:

$$\max_{I_1} \quad 10\sqrt{I_1} - (1+r_1)I_1$$

$$\text{FOC:} \quad \frac{5}{\sqrt{I_1}} - (1+r_1) = 0 \iff I(r_1) = \frac{25}{(1+r_1)^2}$$

- Utility function is $\ln(C_1) + \ln(C_2)$. Therefore, the national saving schedule is of the form:

$$S_1(r_1) = \frac{1}{2}\left[Y_1 - G_1 - \frac{\Pi_2(r_1) - G_2}{1+r_1}\right]$$

- Find $\Pi_2(r_1)$.

$$\Pi_2(r_1) = 10\sqrt{I(r_1)} - (1+r_1)I(r_1) = \frac{25}{1+r_1}$$

- Solve for equilibrium r_1.

$$I(r_1^*) = S_1(r_1^*) \implies r_1^* = 1/4$$

Repeat with new value of G_1

Q2

- Household's budget constraints are as usual. Government does nothing in the first period ($0 = 0$)! In second period, total subsidy paid equals lump-sum tax collected:

$$\tau^I(1 + r_1)I_1 = T_2$$

- Profit maximization for firms:

$$\max_{I_1} \quad 2\sqrt{I_1} - (1 - \tau^I)(1 + r_1)I_1$$

$$\text{FOC:} \quad \frac{1}{\sqrt{I_1}} - (1 - \tau^I)(1 + r_1) = 0 \iff I(r_1) = \frac{1}{[(1 - \tau^I)(1 + r_1)]^2}$$

Profit of firms is disbursed to households in period 2.

$$Y_2 = \Pi_2(r_1) = 2\sqrt{I_1(r_1)} - (1 - \tau^I)(1 + r_1)I(r_1) = \frac{1}{(1 - \tau^I)(1 + r_1)}$$

- Compute T_2 from govt. budget constraint.

$$T_2 = \frac{\tau^I}{(1 - \tau^I)^2(1 + r_1)}$$

Q2

- All saving comes from households, $S_1 = S_1^P$

$$S_1(r_1) = \frac{1}{2}\left[Y_1 - \frac{Y_2 - T_2}{1 + r_1}\right]$$
$$= \frac{1}{2}\left[1 - \frac{1 - 2\tau^I}{[(1 - \tau^I)(1 + r_1)]^2}\right]$$

- Solve for equilibrium.

$$I(r_1^*) = S_1(r_1^*) \implies 1 + r_1^* = \frac{\sqrt{3 - 2\tau^I}}{1 - \tau^I}$$

Compute the remaining quantities

$$I_1^* = I(r_1^*) = \frac{1}{3 - 2\tau^I}, \quad C_1^* = \frac{2 - 2\tau^I}{3 - 2\tau^I}, \quad C_2^* = \frac{2}{\sqrt{3 - 2\tau^I}}$$

Q2

- When $\tau^I = 0$, then

$$r_1^* = 0.732, I_1^* = 0.33, C_1^* = 0.67, C_2^* = 1.15$$

When $\tau^I = 0.5$, then

$$r_1^* = 1.828, I_1^* = 0.5, C_1^* = 0.5, C_2^* = 1.414$$

- **Welfare:** Need to find τ^I that maximizes $\ln(C_1^*) + \ln(C_2^*)$.

$$\begin{aligned} W &= \ln(C_1^*) + \ln(C_2^*) \\ &= \ln(2 - 2\tau^I) - \frac{3}{2}\ln(3 - 2\tau^I) + \ln(2) \end{aligned}$$

Take derivative with respect to τ^I.

$$\frac{dW}{d\tau^I} = \frac{-2}{2 - 2\tau^I} + \frac{3}{3 - 2\tau^I} = \frac{-2\tau^I}{(2 - 2\tau^I)(3 - 2\tau^I)}$$

Since $\tau^I \in [0, 1]$, $\frac{dW}{d\tau^I} \leq 0$. Welfare-maximizing subsidy is zero!

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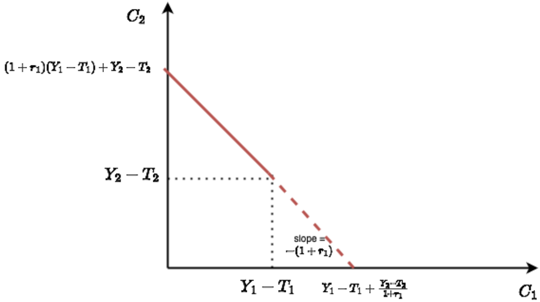
Borrowing constraints

- The household's problem is now subject to a no-borrowing constraint.

$$\max \ln(C_1) + \ln(C_2)$$

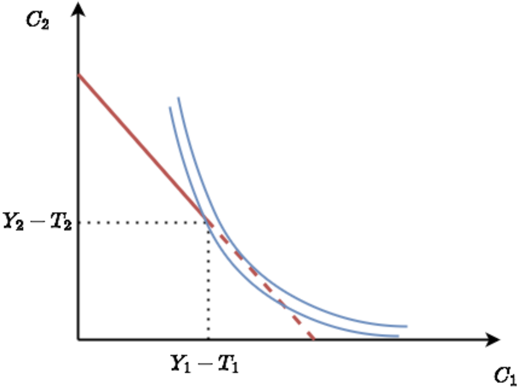
$$\begin{aligned} \text{s.t. } & C_1 + S_1^P = Y_1 - T_1, \\ & C_2 = (1 + r_1)S_1^P + Y_2 - T_2, \\ & S_1^P \geq 0 \end{aligned}$$

- The intertemporal budget constraint is truncated:

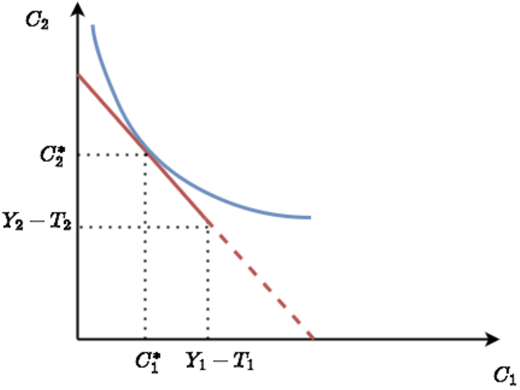


Constraint is binding

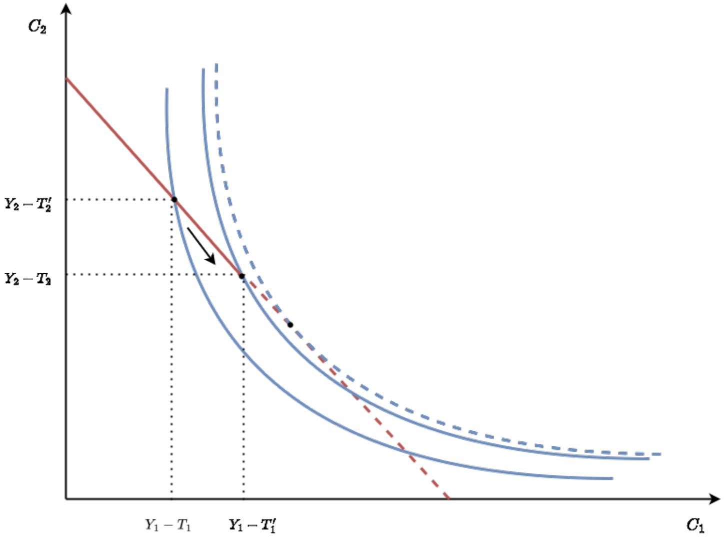
Because of the constraint, the household cannot borrow even if it wants to. They are forced to consume their endowment: $C_1^* = Y_1 - T_1$, $C_2^* = Y_2 - T_2$.



Constraint is not binding

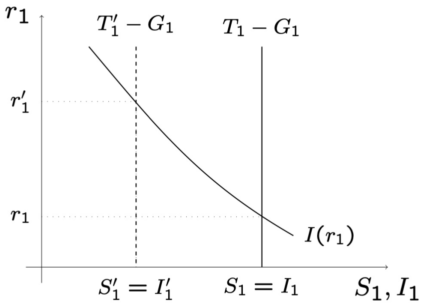


Effect of tax cut



Effect of tax cut

- When households are borrowing-constrained, they consume their endowment, and do not save, i.e. $S_1^p = Y_1 - T_1 - C_1 = 0$.
- All savings come from the government, $S_1 = S_1^g = T_1 - G_1$. Saving is interest-inelastic in this case.
- When there is a tax cut, saving contracts: $\Delta S_1 = \Delta T_1$



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One-period lived consumers

- Suppose the households only live for one period, even though the government lives for two periods.
- For a household that lives in period 1, it is optimal to consume their entire endowment, $C_1^* = Y_1 - T_1$.
- There is no private saving, all saving comes from the government

$$S_1 = S_1^g = T_1 - G_1$$

- Aggregate saving contracts when there is a cut in taxes:

$$\Delta S_1 = \Delta T_1$$

- Cut in taxes causes equilibrium interest rate to rise:

$$I(r_1^*) = T_1 - G_1$$

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Household's problem

- As before, the household wants to maximize lifetime utility
$$\ln(C_1) + \ln(C_2)$$
- But faces the following budget constraints for each period:
$$(1+\tau_1)C_1 + S_1^p = Y_1$$
$$(1+\tau_2)C_2 = \Pi_2 + (1+r_1)S_1^P$$
Instead of a lump-sum tax on income, they are charged a tax for every unit of consumption.
- Eliminating S_1^p, the intertemporal budget constraint is
$$(1+r_1)(1+\tau_1)C_1 + (1+\tau_2)C_2 = (1+r_1)Y_1 + Y_2$$
$$\iff (1+\tau_1)C_1 + \frac{(1+\tau_2)C_2}{(1+r_1)} = Y_1 + \frac{Y_2}{1+r_1}$$
Slope of budget line $= -\frac{(1+\tau_1)(1+r_1)}{(1+\tau_2)}$

Optimal consumption choice

- At optimum, the slope of the budget line equals the slope of the indifference curve.

$$-\frac{(1+\tau_1)(1+r_1)}{(1+\tau_2)} = -\frac{C_2}{C_1}$$

- Rewriting the intertemporal budget constraint for the household:

$$(1+\tau_1)C_1 + \frac{(1+\tau_2)C_2}{(1+r_1)} = Y_1 + \frac{Y_2}{1+r_1}$$

- Solve for C_1^*, C_2^* from these two equations.

$$C_1^* = \frac{1}{2} \cdot \frac{1}{1+\tau_1}\left[Y_1 + \frac{Y_2}{1+r_1}\right], \quad C_2^* = \frac{1}{2} \cdot \frac{1+r_1}{1+\tau_2}\left[Y_1 + \frac{Y_2}{1+r_1}\right]$$

Cobb-Douglas shortcut

Whenever the utility function is of the Cobb-Douglas form

$$\alpha_1 \ln(x_1) + \alpha_2 \ln(x_2), \quad \text{or} \quad x_1^{\alpha_1} x_2^{\alpha_2}$$

And the budget constraint is

$$p_1 x_1 + p_2 x_2 = y,$$

The solution to the maximization problem is

$$x_1^* = \frac{\alpha_1}{\alpha_1 + \alpha_2} \cdot \frac{y}{p_1}, \quad x_2^* = \frac{\alpha_2}{\alpha_1 + \alpha_2} \cdot \frac{y}{p_2}$$

Irrespective of prices, the consumer spends a fixed fraction of their income on each good: $\frac{\alpha_1}{\alpha_1+\alpha_2}$ for good 1, and $\frac{\alpha_2}{\alpha_1+\alpha_2}$ for good 2.

Aggregate saving

- Private saving:

$$S_1^P = Y_1 - (1+\tau_1)C_1^*$$

- Government saving:

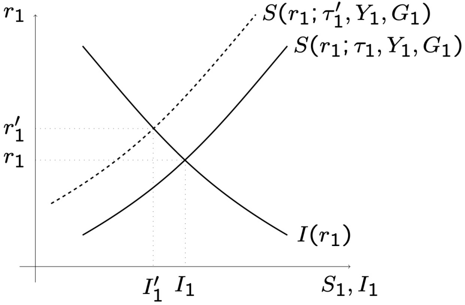
$$S_1^g = \tau_1 C_1^* - G_1$$

- National saving:

$$\begin{aligned} S_1 \equiv S_1^P + S_1^g &= Y_1 - C_1^* - G_1 \\ &= Y_1 - G_1 - \frac{1}{2} \cdot \frac{1}{1+\tau_1}\left[Y_1 + \frac{Y_2}{1+r_1}\right] \\ &= Y_1 - G_1 - \frac{1}{2} \cdot \frac{1}{1+\tau_1}\left[Y_1 + \frac{\Pi_2(r_1)}{1+r_1}\right] \end{aligned}$$

$$S_1 \equiv S\left(\underset{+}{r_1};\ \underset{+}{\tau_1},\ \underset{+}{Y_1},\ \underset{-}{G_1}\right)$$

Effect of a cut in tax rate



- *Note:* The government needs to adjust τ_2 to ensure that it maintains its intertemporal budget constraint.

$$G_1 + \frac{G_2}{1 + r_1} = \tau_1' C_1^* + \frac{\tau_2' C_2^*}{1 + r_1}$$

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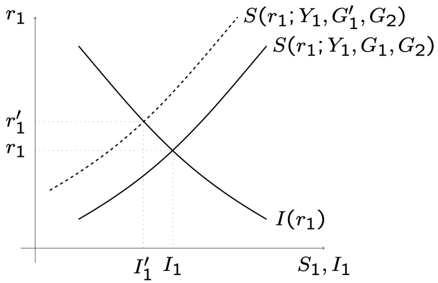
Spending shock in period 1

- Recall the expression for national saving:

$$S_1 = \frac{1}{2}\left[Y_1 - G_1 - \frac{\Pi_2(r_1) - G_2}{1 + r_1}\right]$$

  When G_1 goes up, saving goes down. Less resources available in period 1 $\rightarrow$ less incentive to save.

- Equilibrium interest rate rises to clear up the excess demand:



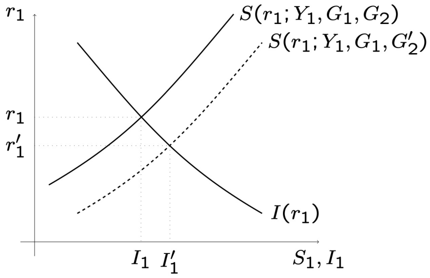
Spending shock in period 2

- Recall the expression for national saving:

$$S_1 = \frac{1}{2}\left[Y_1 - G_1 - \frac{\Pi_2(r_1) - G_2}{1 + r_1}\right]$$

When G_2 goes up, saving goes up. Less resources available in period 2 $\rightarrow$ more incentive to save.

- Equilibrium interest rate falls to clear up the excess supply:



Financing with distortionary taxes

- Suppose the government levies a proportional consumption tax to balance its budget in every period, i.e.

$$G_1 = \tau_1 C_1, \quad G_2 = \tau_2 C_2$$

- Recall optimal period 1 consumption under a proportional tax:

$$C_1 = \frac{1}{2} \cdot \frac{1}{1+\tau_1} \left[Y_1 - G_1 + \frac{\Pi_2(r_1) - G_2}{1+r_1} \right]$$

- The saving schedule does not depend on G_1, G_2.

$$\begin{aligned} S_1 &= Y_1 - (C_1 + G_1) \\ &= \frac{1}{2} \left[Y_1 - \frac{\Pi_2(r_1)}{1+r_1} \right] \end{aligned}$$

  Spending shocks have no effect on interest rate or investment.

- $G_1 \uparrow \implies C_1 \downarrow$, since $C_1 = Y_1 - I_1 - G_1$